Review of CS

Simple idea: (I) and (II) equivalent under some conditions on measurement matrix A. $||x||_0$, a measure of the number of nonzeros, not a norm. $||x||_1$, sum of absolute values of components, convex norm.

$$(I) Ax = b, \min ||x||_0$$

$$(II) Ax = b, \min ||x||_1 \rightarrow Ax \approx b, \min ||x||_1 \rightarrow min||Ax - b|| + \lambda ||x||_1$$

RIP condition: $(1-\sigma_s)||y||_2^2 \leq ||A_s y||_2^2 \leq (1+\sigma_s)||y||_2^2$

For every s columned submatrix A_s of A for sparse vector y. Why unlikely for real world: any not so well conditioned matrix would have A_sy =0. What this means: (II) would not recover the sparsest possible solution, but would still yield a sparse solution. ML can be used to predict the approximate regions of support of x (e.g. in ROMP-like methods), and relax the needed condition on A.

Some algorithms based on CS for imaging:
(1) min gradient reconstruction in transformed domain, (2) matrix completion-based methods, (3) single image super res.

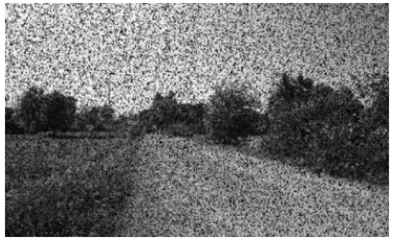
REGULARIZED ORTHOGONAL MATCHING PURSUIT (ROMP)

INPUT: Measurement vector $x \in \mathbb{R}^N$ and sparsity level n
OUTPUT: Index set $I \subset \{1, \dots, d\}$
Initialize: Let the index set $I = \emptyset$ and the residual $r = x$.
Repeat the following steps until $r = 0$:
Identify: Choose a set J of the n biggest coordinates in magnitude of the observation vector $u = \Phi^* r$, or all of its nonzero coordinates, whichever set is smaller.
Regularize: Among all subsets $J_0 \subset J$ with comparable coordinates:

$$|u(i)| \leq 2|u(j)| \quad \text{for all } i, j \in J_0,$$

choose J_0 with the maximal energy $\|u|_{J_0}\|_2$.
Update: Add the set J_0 to the index set: $I \leftarrow I \cup J_0$, and update the residual:

$$y = \operatorname*{argmin}_{z \in \mathbb{R}^I} \|x - \Phi z\|_2; \qquad r = x - \Phi y.$$



Sergey Voronin, 8.2019

Computational imaging

Hardware + software solutions outperform all-optics alone.

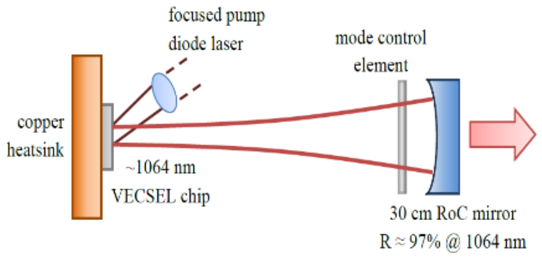
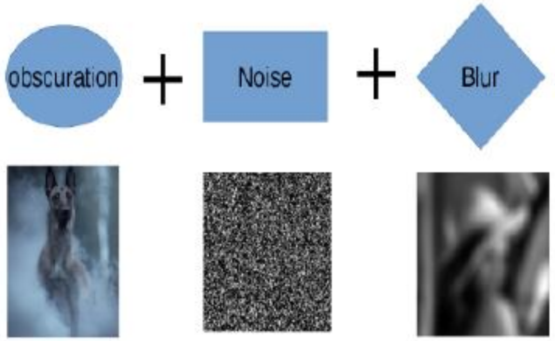
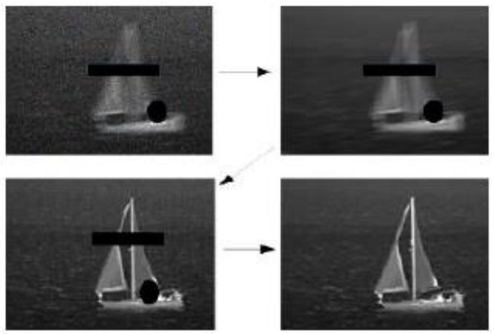


Illustration of VECSEL system from University of Arizona. Very promising technology for it's small size and long distance reach.

EO/IR system output susceptible to atmospheric and illumination conditions. General degradation model takes into account obscuration, noise, and blur. Promising CNN based techniques can address obscuration artifacts better than optimization alone.

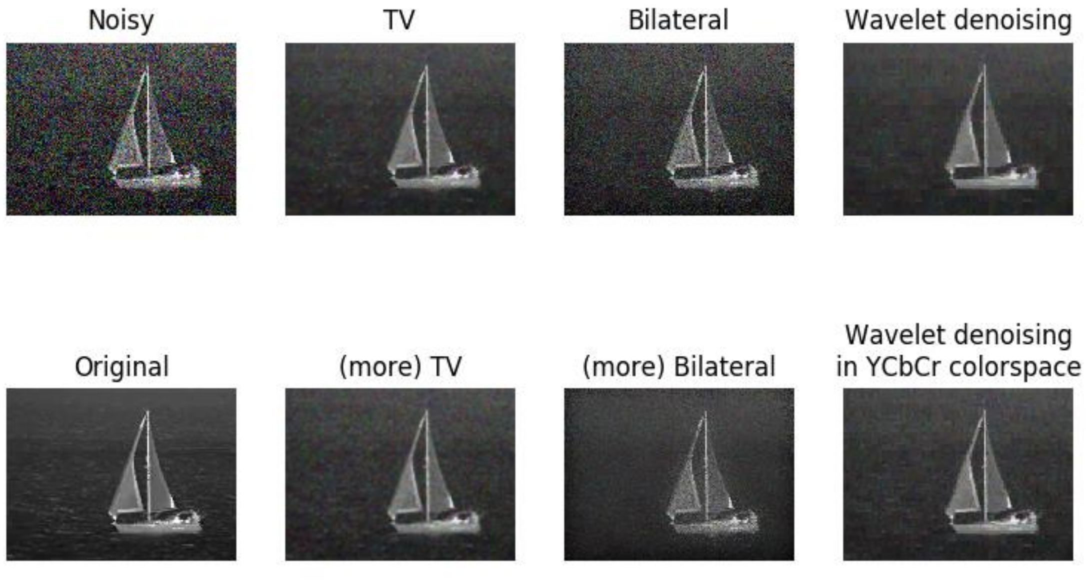




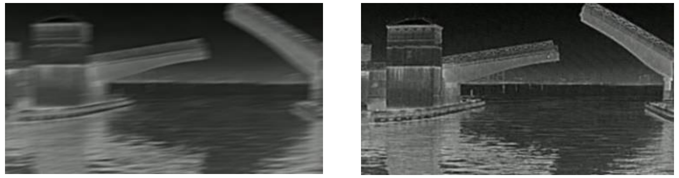
Deep bilateral learning ('17)

Image enhancement

Noise: multi-resolution image decomposition + thresholding; optimization based e.g. TV-norm min. ML methods for denoising.



Blur: Wiener filtering + deconvolution.



$$g(x,y) = h(x,y) \star f(x,y) + n(x,y),$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = W(u,v)G(u,v)$$

$$W(u,v) = \frac{H^{\{*\}}(u,v)}{|H(u,v)|^2 + K(u,v)}$$

Image enhancement

Partial obscuration: matrix-completion methods (e.g. singular value threhsolding), sparse gradient minimization in transformed domain (based on CS).

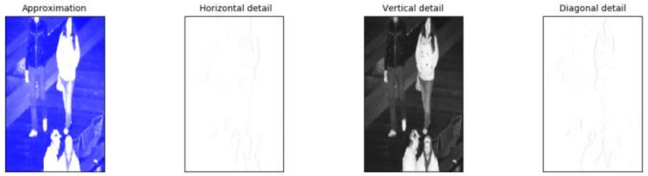
$$x_a^{(0)}(n) = \begin{cases} 0 & \text{for missing samples, } n \in \mathbb{N}_Q \\ x(n) & \text{for available samples, } n \in \mathbb{N}_A \end{cases}$$

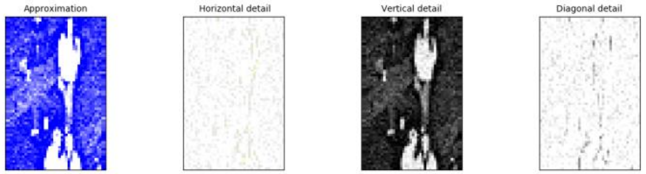
$$\min \quad \|\mathbf{X}_a\|_1$$
$$\text{subject to} \quad x_a^{(m)}(n) = x(n) \quad \text{for } n \in \mathbb{N}_A$$

$$\begin{aligned} X_a^{+}(k) &\leftarrow \mathrm{DFT}\{x_a^{(m)}(n) + \Delta\delta(n - n_i)\} \\ X_a^{-}(k) &\leftarrow \mathrm{DFT}\{x_a^{(m)}(n) - \Delta\delta(n - n_i)\} \\ g^{(m)}(n_i) &\leftarrow (1/N) \textstyle\sum_{k=0}^{N-1} |X_a^{+}(k)| - |X_a^{-}(k)| \end{aligned}$$

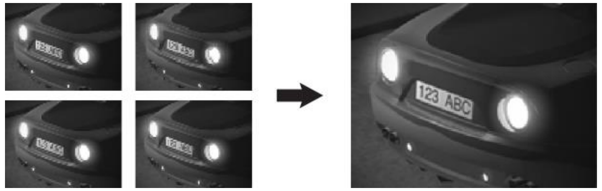
This class of methods [Stankovic, et. al.] essentially modifies pixel values to connect straight lines over transformed image patches.

Residual learning with CNN: train on original and partially obscured image sets in MRA setting to enable automatic pixel correction.



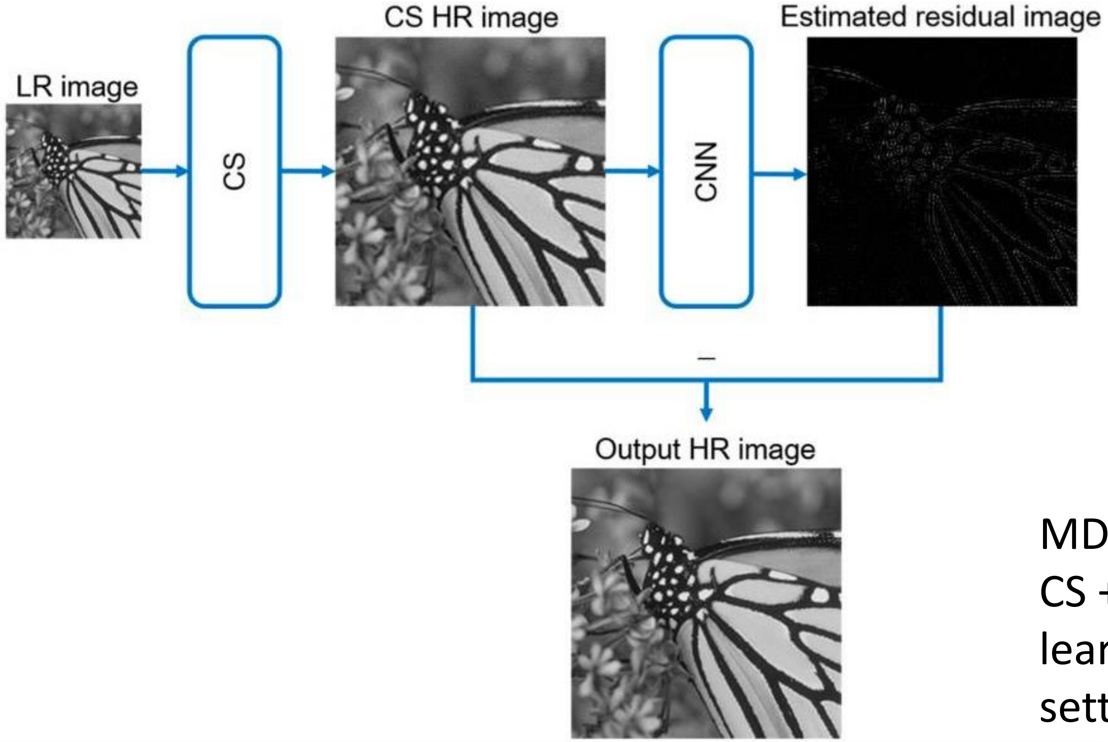


Single image super-resolution: sparsity based (CS) and learning based methods. Basic idea is that patch of HR image is sparse under suitable transform.



Opt. based upscaling from multiple images with assumed degradation factors.

$$\bar{X} = argmin_x \left\{ \sum_{\{k=1\}}^{M} ||D_k H_k F_k X - Y_k||_2^2 + \lambda R(X) \right\}$$



MDPI ’18
CS + CNN based residual learning (extend to MRA setting).